Partial Differential Equations in General Relativity

Book reviews

The Wraparound Universe
Jean-Pierre Luminet
2008 A K Peters
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I thoroughly enjoyed reading this book. It is written in clear language supplemented with many very helpful photographs and drawings. I like the structure of the book, which is a collection of 45 rather short chapters that make it easier for the reader to read it at his/her own speed.

The main aim of the author is to interest the reader in cosmology and to convey to him/her the amazing progress that has been made in recent years in our understanding of the universe, its shape and its future. However, even to formulate this problem and to describe some recent work in this field, the author has to explain to the reader many concepts from mathematics and physics. Jean-Pierre Luminet, in addition to being a well known astrophysicist, is also a very gifted writer and so he manages to do this very successfully. In fact the book contains very few formulae and most of the explanations are given in terms of a written narrative supplemented by drawings. The author is also extremely skillful in finding and then using appropriate analogies. The required ideas from mathematics, and topology in particular, present a further aim of the book—to explain to the interested reader the beautiful world of topology and its relevance to the description of the real world. Here, again, he succeeds very impressively.

The central claim of the book is as follows: instead of a simple topology, the Universe may have a multiply-connected topology—hence 'wrapped around'; in consequence, it may be much smaller than is usually assumed. If this is so some of the galaxies we see are not real galaxies, but only images of a smaller number of genuine galaxies. The author then discusses possible topologies, and finally chooses the 'dodecahedral' one. A large part of the book is dedicated to showing how this hypothesis can be tested, and what the most recent data on the cosmic background radiation from the WMAP satellite say about this issue (they are inconclusive). Jean-Pierre Luminet's suggestions disagree with the standard inflationary model, which uses the same data to argue that the Universe is spatially flat, and so infinite.

The author is also scrupulous in apportioning priorities. As he explains in detail in several historical sections, the standard cosmological equations (normally called Robertson Walker, or Friedmann Robertson Walker equations) were first written by Lemaitre and Friedmann—hence in the book the cosmological models which use them are always referred to as Friedmann–Lemaitre models. Similarly, the Doppler effect becomes the Doppler–Fizeau effect and Hubble's law is entitled Hubble–Lemaitre.

I also liked the sections of the book in which the author shows how the same ideas in different historic or geographic conditions have had different impacts on the development of science; some were ignored, some misunderstood and some considered more seriously than they deserved.

All in all, 'The Wraparound Universe' is a great general-audience book and I recommend it unreservedly.

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Tribology on the Small Scale: A Bottom Up Approach to Friction, Lubrication, and Wear
C Mathew Mate
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Friction, lubrication and wear interactions between materials make considerable differences to how efficient our engines are, whether or not we ski downhill faster than others, or whether the shoes that we are wearing give us sufficient grip to successfully navigate the marble floors of buildings. Traditionally, tribologists have focussed on the macroscopic issues of tribological problems, looking at the design of components, the viscosity of oils and the mechanical properties of surfaces to understand how components interact to give the desired friction and wear properties. However, in the last twenty years there has been an increasing realization that the processes that are controlling these macroscopic interactions are determined by

what happens on the atomic and microscopic scale. Further, with the advent of nano- and micro-electro mechanical systems (NEMs and MEMs), macroscopic scale tribological interactions do not influence the tribology of these devices in the same way, and capillary forces and van der Waal's forces play an increased role in determining whether these devices function successfully.

This book aims to fill a gap in the area of tribology textbooks by addressing the important advances that have been made in our understanding of the science of nano- and micro-scale tribological interactions. The book is aimed at advanced undergraduate and graduate level students on engineering programmes, academics and scientists interested in atomic and microscopic scale tribological interactions, and engineers and scientists who are not tribologists *per se* but work in technologies (such as NEMs/MEMs) where tribology is of importance. Whilst the target audience appears to be largely engineers, the book should have wider appeal to physicists, chemists and modellers with interests in tribological interactions.

The book consists of twelve chapters with an introduction to the general significance of tribology and a brief history of modern tribology, followed by more detailed coverage of characterization and quantification of surface roughness. There is then a discussion of the mechanical properties of surfaces, and an introduction to contact mechanics. This follows a similar structure to traditional tribology textbooks but there are some nice examples and illustrations of how this relates to small scale tribology, with reference to recording heads on laser textured disk surfaces for example. The origins of friction are then discussed, with a detailed section on stick-slip interactions which are particularly significant in tribological interactions at the small scale.

Chapters 5–8 then deviate from the more traditional tribology textbooks and cover surface energies and capillary forces, surface forces and their physical origins, and the measurement of these forces by the surface force apparatus and atomic force microscope. Surface forces at the small scale and capillary forces are extremely important in the successful functioning of small scale nano- or micro-electro mechanical systems, and there is a good discussion of the origin of these forces and how they can be understood, measured and controlled.

The final chapters are devoted to lubrication, and atomistic origins of friction and wear. Traditional lubrication theories are initially out-lined followed by detailed examples of boundary lubrication and capillary forces in tribology at the micro-scale. There are some nice examples of the importance of lubricant chemistry on sliding forces.

Overall I found this book to be well-written and very readable with some very nice examples of why all this fundamental background is of importance in practical applications. The book is well-presented and it should be accessible to its target audience, particularly since the cost is reasonable. Each chapter ends with a set of selected references to allow more detailed study of particular topics if desired. There is a comprehensive index at the end of the book. I will recommend it to my students on courses on tribology and surface engineering.

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Partial Differential Equations in General Relativity
Alan D Rendall
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Although many books on general relativity contain an overview of the relevant background material from differential geometry, very little attention is usually paid to background material from the theory of differential equations. This is understandable in a first course on relativity but it often limits the kinds of problems that can be studied rigorously.

Einstein's field equations lie at the heart of general relativity. They are a system of partial differential equations (PDEs) relating the curvature of spacetime to properties of matter. A central part of most problems in general relativity is to extract information about solutions of these equations. Most standard texts achieve this by studying exact solutions or numerical and analytical approximations. In the book under review, Alan Rendall emphasises the role of rigorous qualitative methods in general relativity. There has long been a need for such a book, giving a broad overview of the relevant background from the theory of partial differential equations, and not just from differential geometry. It should be noted that the book also covers the basic theory of ordinary differential equations.

Although there are many good books on the rigorous theory of PDEs, methods related to the Einstein equations deserve special attention, not only because of the complexity and importance of these equations, but because these equations do not fit into any of the standard classes of equations (elliptic, parabolic, hyperbolic) that one typically encounters in a course on PDEs. Even specifying

exactly what ones means by a Cauchy problem in general relativity requires considerable care. The main problem here is that the manifold on which the solution is defined is determined by the solution itself. This means that one does not simply define data on a submanifold.

Rendall's book gives a good overview of applications and results from the qualitative theory of PDEs to general relativity. It would be impossible to give detailed proofs of the main results in a self-contained book of reasonable length. Instead, the author concentrates on providing key definitions together with their motivations and explaining the main results, tools and difficulties for each topic. There is a section at the end of each chapter which points the reader to appropriate literature for further details. In this way, Rendall manages to describe the central issues concerning many subjects.

Each of the twelve chapters (except for one on functional analysis) contains an important application to general relativity. For example, the chapter on ODEs discusses Bianchi spacetimes and the Einstein constraint equations are discussed in the chapter on elliptic equations. In the chapter on hyperbolic equations, the Einstein dust system is considered in the context of Leray hyperbolicity

and Gowdy spacetimes are analysed in the section on Fuchsian methods. The book concludes with four chapters purely on applications to general relativity, namely The Cauchy problem for the Einstein equations, Global results, The Einstein–Vlasov system and The Einstein-scalar field systems.

On reading this book, someone with a basic understanding of relativity could rapidly develop a picture, painted in broad brush strokes, of the main problems and tools in the area. It would be particularly useful for someone, such as a graduate student, just entering the field, or for someone who wants a general idea of the main issues. For those who want to go further, a lot more reading will be necessary but the author has sign-posted appropriate entry points to the literature throughout the book. Ultimately, this is a very technical subject and this book can only provide an overview. I believe that Alan Rendall's book is a valuable contribution to the field of mathematical relativity.

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